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Time-Optimal Orbit Transfer Trajectory for Solar Sail Spacecraft

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This paper considers the problem of optimally controlling the sail steering angle of a solar sail spacecraft (a spaceship moving under the influence of solar radiation pressure) so as to execute a minimum-time coplanar orbit transfer from the mean orbital distance of Earth to the mean Martian orbital distance around the sun. This problem has been modeled as a free-terminal time-optimal control problem with an unbounded control variable and with state variable equality constraints at the final time. It has been solved by the penalty function approach using the conjugate gradient algorithm. Detailed computational results regarding the convergence of the optimal control, the state trajectories, and the norm of the gradient trajectory, along with the minimization of the performance functional, are presented. In general, the optimal solution is found to be compatible with that obtained by some of the earlier investigators of this problem. In conclusion, the optimized orbit transfer time of the solar sail spacecraft is compared with that of an ionic propulsion system.

Nomenclature

a = solar gravitational acceleration at the mean orbital distance of Earth
f = a vector mapping function
g_u = gradient
H = Hamiltonian
J = augmented performance functional
L = integrand of the performance functional
t = normalized time
u = solar sail steering angle (control variable) measured from radial orientation
u^0 = nominal control
u^* = optimal control
V = inertial velocity of spacecraft
x_1 = normalized heliocentric radial displacement
x_2 = normalized radial velocity
x_3 = normalized circumferential velocity
x_4 = heliocentric angle
ϕ = penalty function
$\lambda_1, \lambda_2, \lambda_3$ = costates corresponding to states x_1, x_2, and x_3
α = acceleration of spaceship due to solar radiation pressure alone on radially oriented solar sail at the mean orbital distance of Earth
β = nondimensional solar radiation pressure-induced acceleration parameter of spaceship due to radially oriented solar sail at the mean orbital distance of Earth
γ = spiral angle (inertial flight path angle) measured from local horizon to the inertial velocity vector

Subscripts

0 = initial time
f = terminal time
T = transpose
$(\dot{\ })$ = derivative with respect to time

Introduction

THE potential capabilities of solar sail propulsion have been studied by several investigators. It is well known that a solar sailing ship needs no fuel or propellant, has no powerplant on board ship, and has no waste-heat disposal problems, and hence it may cost significantly less than the electric propulsion vehicles. Garwin[1] states that the solar sail is perhaps more powerful and less difficult than many often-cited competing schemes. Several investigations dealing with interplanetary trajectories of the solar sail have been made through the analytical and the indirect variational approaches, as well as from the standpoint of optimal control.

Tsu[2] examined the characteristics of the solar sail in some detail. He made a simplification by neglecting the time derivative of the radial velocity in the equations governing the dynamics of the solar sail spacecraft. Thus, he solved the approximate equations of motion and showed that the spaceship moves along logarithmic spiral trajectories. For various numerical values of the sail acceleration parameter, he optimized the sail tilt angle of the spaceship corresponding to minimum travel time along appropriate logarithmic spirals to mean Martian orbital distance. He also compared the travel time of the solar sail propulsion system with that of an ionic propulsion system originally investigated by Stuhlinger.[3] London[4] derived some exact solutions of the equations of motion of the spacecraft with a constant sail setting for various spiral angles. The analysis he used is analogous to that of Bacon[5] in which Bacon stipulated a logarithmic spiral trajectory for a vehicle in an inverse-square gravitational field and then showed that in the case of tangential thrust such a trajectory would be obtained if the thrust-to-mass ratio of the vehicle were varied as the inverse of the square of the radial distance. By employing the logarithmic spiral solution, London derived the expression connecting the spiral angle and the sail tilt angle of the spaceship. He also demonstrated that for small values of the spiral angle, the solution reduces to the approximate solution given by Tsu. London further derived the general expression for the travel time along logarithmic spiral trajectories in terms of the spiral angle and the sail tilt angle, and obtained the optimum spiral angles and the necessary sail settings as a function of the sail acceleration parameter, corresponding to minimum travel time. He also showed that the solar sail trajectories would be conic sections if the sail setting were always radial, and computed the travel time with a radial sail setting as a function of the sail acceleration parameter to mean Martian orbital distance for cotangential transfer from Earth's heliocentric orbit. In their analytical approach for obtaining optimum travel time solutions, Tsu and London disregarded the two-point boundary conditions.

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Zhukov and Lebedev,[6] by employing the indirect variational approach due to Rozonoer,[7] formulated the problem of minimum-time interplanetary orbit transfer of the solar sail as a two-point boundary value problem, thus incorporating the two-point boundary conditions. With the aid of Pontryagin's maximum principle and by maximizing the Hamiltonian function, they derived an expression uniquely specifying the solar sail steering angle in terms of the adjoint variables of the system. Thus, they reduced the solution of the two-point boundary value problem by Newton's method to a multiple solution of Cauchy's problem, and obtained the optimal travel time solutions to mean Martian orbital distance for various values of the sail acceleration parameter by the indirect approach.

Kelley[8,9] formulated this problem from the viewpoint of optimal control, and obtained the minimum travel time solution using one of the direct methods of the calculus of variations, namely the method of gradients or "the method of steepest descent," which offers circumvention of the two-point boundary value difficulty encountered when the classical indirect approach (namely, numerical integration of the Euler-Lagrange equations) is used. He reported detailed computational results. In the present work, the conjugate gradient algorithm,[10,11] which gives better convergence than the gradient methods, has been used. Detailed computational results regarding the convergence of the optimal control, the state trajectories, and the norm of the gradient trajectory, along with the minimization of the performance functional, are reported. The optimal solution is compared with that obtained by Zhukov and Lebedev, as well as with the gradient and Euler solutions reported by Kelley. Finally, the optimized orbit transfer time of the solar sail spacecraft is compared with that of an ionic propulsion system reported by Kelley and several other investigators.[12-14]

Dynamics of the Spacecraft

The orbits of Earth and Mars are approximated as concentric circles within the same plane. The ship's motion along the trajectory around the sun is treated as a two-body problem involving the sun and the ship. The sun and the ship are modeled as point masses. Only the trajectory around the sun is considered. With these assumptions, and with reference to a heliocentric inertial frame given in Fig. 1, the normalized dynamics of the solar sail spacecraft is given by

Radial velocity:

$$\dot{x}_1 = x_2 \tag{1}$$

Radial acceleration:

$$\dot{x}_2 = x_3^2/x_1 - 1/x_1^2 + (\beta/x_1^2)\,|\cos^3 u| \tag{2}$$

Circumferential acceleration:

$$\dot{x}_3 = -x_2 x_3/x_1 - (\beta/x_1^2)\,\text{sin}u\cos^2 u \tag{3}$$

Circumferential angular velocity:

$$\dot{x}_4 = x_3/x_1 \tag{4}$$

where $u(t)$ is the solar sail steering angle treated as the control variable measured from radial orientation, and $x_1(t)$, $x_2(t)$, $x_3(t)$, and $x_4(t)$ are the states of the dynamic system. Since the heliocentric angle x_4 does not appear in Eqs. (1-3), nor in the statements of the boundary conditions to be considered, Eq. (4) may be ignored for the purposes of optimization. This amounts to an assumption that the terminal matching of the heliocentric angles of the vehicle and the target planet is accomplished by the selection of launch time. Using other terminology, this states that a proper "launch window" has to be selected in order to accomplish a successful trajectory.

The normalized initial and boundary conditions are

$$x_1(0) = x_{1_0} \quad x_2(0) = x_{2_0} \quad x_3(0) = x_{3_0} \tag{5}$$

$$x_1(t_f) = x_{1_f} \quad x_2(t_f) = x_{2_f} \quad x_3(t_f) = x_{3_f} \tag{6}$$

where $x_{1_0} = 1.0$, $x_{2_0} = 0.0$, and $x_{3_0} = 1.0$ (corresponding to motion in the Earth's heliocentric orbit) and $x_{1_f} = 1.525$, $x_{2_f} = 0.0$, and $x_{3_f} = 0.8098$ (corresponding to arrival at the heliocentric orbit of Mars).

Two numerical values of the sail acceleration from Tsu[2] have been employed in the computations, namely $\alpha = \alpha_1 = 0.1$ cm/s^2 and $\alpha = \alpha_2 = 0.2$ cm/s^2, and hence, the corresponding nondimensional solar radiation pressure-induced acceleration parameters of the sail are $\beta = \beta_1 = \alpha_1/a$ and $\beta = \beta_2 = \alpha_2/a$, where $a = 0.592$ cm/s^2. Thus, $\beta_1 = 0.16892$ and $\beta_2 = 0.33784$.

Statement of the Problem

The problem is to determine an optimal control $u^*(t)$ over $0 \le t \le t_f$, with the given initial conditions [Eq. (5)], such that the final time t_f is minimized subject to the differential constraints [Eqs. (1-3)] and the boundary conditions [Eq. (6)].

Application of the Conjugate Gradient Algorithm for Free-Terminal Time

The conjugate gradient algorithm for free-terminal time is applied to solve this problem. Boundary conditions [Eq. (6)] are included in the performance index via penalty functions. Therefore, the performance index for this problem takes the form

$$J = \int_0^{t_f} L\mathrm{d}t + \{\varphi[x(t_f)]\} \tag{7}$$

$$= t_f + \tfrac{1}{2} S_{11}[x_1(t_f) - x_{1_f}]^2 + \tfrac{1}{2} S_{22}[x_2(t_f) - x_{2_f}]^2 + \tfrac{1}{2} S_{33}[x_3(t_f) - x_{3_f}]^2 \tag{8}$$

where S_{11}, S_{22}, and S_{33} are the terminal penalties. The Hamiltonian function for this problem is given by

$$H = L + \lambda^T f \tag{9}$$

$$= 1 + \lambda_1 x_2 + \lambda_2[x_3^2/x_1 - 1/x_1^2 + (\beta/x_1^2)\,|\cos^3 u|] - \lambda_3[x_2 x_3/x_1 + (\beta/x_1^2)\,\text{sin}u\cos^2 u] \tag{10}$$

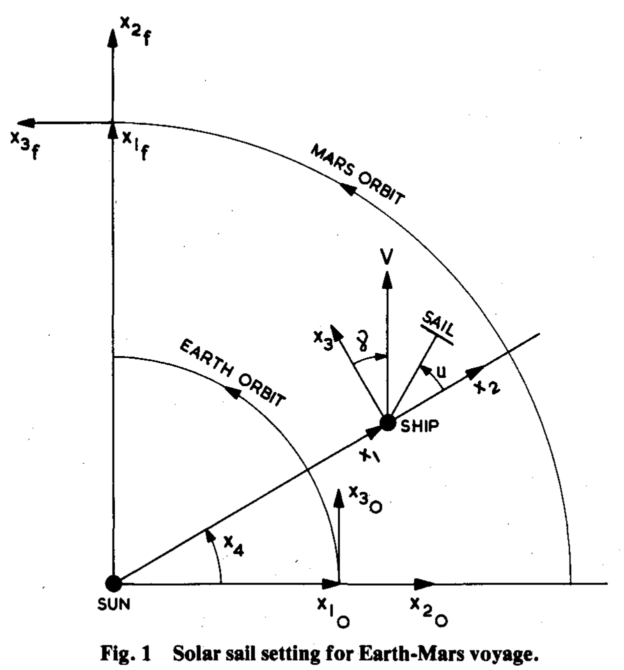

Fig. 1 Solar sail setting for Earth-Mars voyage.

where λ_1, λ_2, and λ_3 are the costates of the dynamic system. Application of Pontryagin's maximum principle[15] leads to the following equations:

$$\dot{\lambda}_1 = -\partial H/\partial x_1 \tag{11}$$

$$= \lambda_2 [x_3^2/x_1^2 - 2/x_1^3 + (2\beta/x_1^3)\,|\cos^3 u|\,] - \lambda_3 [x_2 x_3/x_1^2 + (2\beta/x_1^3)\sin u \cos^2 u] \tag{12}$$

$$\lambda_1(t_f) = \partial\varphi[x(t_f)]/\partial x_1(t_f) \tag{13}$$

$$= S_{11}[x_1(t_f) - x_{1_f}] \tag{14}$$

Similarly,

$$\dot{\lambda}_2 = -\lambda_1 + \lambda_3(x_3/x_1) \tag{15}$$

$$\lambda_2(t_f) = S_{22}[x_2(t_f) - x_{2_f}] \tag{16}$$

and

$$\dot{\lambda}_3 = -2\lambda_2(x_3/x_1) + \lambda_3(x_2/x_1) \tag{17}$$

$$\lambda_3(t_f) = S_{33}[x_3(t_f) - x_{3_f}] \tag{18}$$

The gradient is given by

$$g_u = \partial H/\partial u \tag{19}$$

$$= -[\lambda_2(3\beta/x_1^2)\cos^2 u \sin u\, \mathrm{sgn}(\cos^3 u) + \lambda_3(\beta/x_1^2)(\cos^3 u - 2\sin^2 u \cos u)] \tag{20}$$

The criterion for determining the final time t_f is obtained via the conditions on the Hamiltonian (for a free-terminal time problem) at $t = t_f$.[16] For this problem, it turns out that t_f must satisfy the following equation:

$$\mathrm{d}J/\mathrm{d}t_f - 1 = S_{11}[x_1(t_f) - x_{1_f}]\dot{x}_1(t_f) + S_{22}[x_2(t_f) - x_{2_f}]\dot{x}_2(t_f) + S_{33}[x_3(t_f) - x_{3_f}]\dot{x}_3(t_f) = 0 \tag{21}$$

where the various derivatives have already been defined.

Nominal Control Programs

Corresponding to the nondimensional solar sail acceleration parameter β_1, the assumed nominal sail steering program $u^\circ(t)$ is given by

$$u^\circ(t) = -10 \text{ deg} \quad \text{for} \quad 0 \le t \le 0.84$$

$$u^\circ(t) = -70 \text{ deg} \quad \text{for} \quad 0.84 < t \le 7.0 \tag{22}$$

and for β_2

$$u^\circ(t) = -10 \text{ deg for} \quad 0 \le t \le 0.84$$

$$u^\circ(t) = -70 \text{ deg for} \quad 0.84 < t \le 6.0 \tag{23}$$

Method of Solution

The solution to the problem is started by assuming a nominal control $u^\circ(t)$ from Eq. (22) or (23), from which the initial trajectories $x_1^\circ(t)$, $x_2^\circ(t)$, and $x_3^\circ(t)$ are determined via Eqs. (1-3), with the initial condition from Eq. (5) and the cost function derivative $\mathrm{d}J/\mathrm{d}t_f$ from Eq. (21). When $(\mathrm{d}J/\mathrm{d}t_f - 1)$ goes through zero (and if the second derivative is positive), the first iteration final time has been determined. With this final time and the corresponding values of $x_1^\circ(t)$, $x_2^\circ(t)$, and $x_3^\circ(t)$, the adjoint Eqs. (12), (15), and (17) are solved backward from t_f to zero with the terminal conditions [Eqs. (14, 16, and 18)]. Now, there is enough data to go through the first iteration of the conjugate gradient algorithm. The procedure is repeated until convergence is obtained.

As required by the conjugate gradient algorithm, in every iteration the corresponding approximation to the optimal control program was updated using the distance traveled along the corresponding search direction, this distance being determined by the one-dimensional minimization of the performance functional employing the Quadratic-convergent search technique due to Powell[17] as amended by Zangwill.[18]

Numerical Results

Computations were performed in double-precision arithmetic for a terminal penalty of $S_{11} = S_{22} = S_{33} = 30{,}000$. For the sail acceleration parameter β_1, the convergence is shown in Tables 1 and 2; and for β_2, the convergence is shown in Tables 2 and 3. It is seen from Tables 1 and 3 that the conjugate gradient method essentially converged to the optimal solution in 50 iterations in both the cases (as the difference between successive values of the augmented performance functional is almost zero, and as the norm of the gradients is also tending toward zero), given a nominal control $u^\circ(t)$ as shown in Eqs. (22) and (23), respectively. For both the cases, it is observed from Table 2 that, corresponding to the 50th iteration, the norm of error in the computed state vector at terminal time, where,

$$\text{norm} = \{[x_{1_f} - x_1(t_f)]^2 + [x_{2_f} - x_2(t_f)]^2 + [x_{3_f} - x_3(t_f)]^2\}^{1/2}$$

is negligibly small, and the absolute error in the performance functional due to the incorporation of terminal penalties [namely $(J - t_f)$] is almost zero. For the acceleration parameter β_1, the achieved minimum time is 7.66 which corresponds to 445.5 days; and for β_2, the achieved minimum time is 6.11 which corresponds to 355.4 days. For both the cases, graphical results are presented in Figs. 2-8. Figure 2 depicts the temporal evolution of the optimal sail steering program, while Figs. 3-6 depict the evolution of the state trajectories corresponding to the optimal control. Figures 7 and 8 show the minimization of the performance functional and the time-optimal orbit transfer trajectory, respectively. The average computation time per iteration in double-precision arithmetic, using the IBM System 360/Model 44 computer, was nearly 20 s.

Discussion

With reference to Table 4, it may be seen that the travel time solutions obtained in the present analysis are generally compatible with those arrived at by Zhukov and Lebedev, as well as by Kelley. By the indirect variational method, Zhukov and Lebedev achieved the smallest final time of 405 days corresponding to a sail acceleration of 0.1 cm/s^2; also for a sail acceleration of 0.2 cm/s^2 they obtained a final time of 322 days for travel to mean Martian orbital distance. Though the travel times estimated by them are less than the corresponding values obtained in the present work, it may be seen from Table 2 (corresponding to 50th iteration) that the norm of error in the computed state vector at terminal time and the absolute error in the augmented performance functional are almost zero, which amply demonstrate the extent to which the terminal conditions on the states have been satisfied within the achievable accuracy limits of double-precision arithmetic. On the other hand, Zhukov and Lebedev did not indicate the extent to which the terminal conditions on the states have been satisfied in their analysis.

Employing a numerical value of 0.1 cm/s^2 for the sail acceleration parameter, Kelley has achieved a final time of 7.09 (412.5 days) in a lesser number of iterations using the

Table 1 Minimization of performance functional for β_1

Iteration no.	Terminal time t_f	Performance functional J	Norm of gradients (g_i, g_j)	Computed states at terminal time x_1	x_2	x_3
1	7.00	2666.87900	0.70465×10^{8}	0.114135×10^{1}	0.255448×10^{-1}	0.981507×10^{0}
10	7.67	8.71054	0.90044×10^{3}	0.152281×10^{1}	0.298288×10^{-2}	0.802338×10^{0}
20	7.66	7.66446	0.26791×10^{1}	0.152516×10^{1}	-0.173424×10^{-3}	0.810292×10^{0}
30	7.66	7.66000	0.13325×10^{-3}	0.152500×10^{1}	0.122063×10^{-5}	0.809801×10^{0}
40	7.66	7.66000	0.70693×10^{-10}	0.152500×10^{1}	-0.589932×10^{-9}	0.809800×10^{0}
50	7.66	7.66000	0.16507×10^{-11}	0.152500×10^{1}	0.897817×10^{-10}	0.809800×10^{0}

Terminal penalties: $S_{11}=S_{22}=S_{33}=30{,}000$.
Prescribed states at terminal time: $x_{1_f}=1.525, x_{2_f}=0.0, x_{3_f}=0.8098$.

Table 2 Error in augmented performance functional and terminal state vector

Iteration no.	Absolute error in augmented performance functional, $J-t_f$: For β_1	For β_2	Norm of error in computed state vector at terminal time: For β_1	For β_2
1	0.265988×10^{4}	0.104641×10^{3}	0.421100×10^{0}	0.835230×10^{-1}
10	0.104054×10^{1}	0.607686×10^{0}	0.832883×10^{-2}	0.636494×10^{-2}
20	0.446175×10^{-2}	0.126061×10^{0}	0.545390×10^{-3}	0.289897×10^{-2}
30	0.292380×10^{-7}	0.169746×10^{-3}	0.139614×10^{-5}	0.106379×10^{-3}
40	0.643929×10^{-14}	0.113966×10^{-6}	0.663849×10^{-9}	0.275640×10^{-5}
50	0.888178×10^{-15}	0.215383×10^{-13}	0.273328×10^{-9}	0.120547×10^{-8}

Table 3 Minimization of performance functional for β_2

Iteration no.	Terminal time t_f	Performance functional J	Norm of gradients (g_i, g_j)	Computed states at terminal time x_1	x_2	x_3
1	6.00	110.64130	0.74771×10^{6}	0.151463×10^{1}	-0.764225×10^{-1}	0.777734×10^{0}
10	6.03	6.63769	0.67368×10^{5}	0.151954×10^{1}	0.146671×10^{-3}	0.806532×10^{0}
20	6.11	6.23606	0.47041×10^{4}	0.152315×10^{1}	0.130221×10^{-2}	0.807988×10^{0}
30	6.11	6.11017	0.15520×10^{1}	0.152494×10^{1}	0.395642×10^{-4}	0.809719×10^{0}
40	6.11	6.11000	0.10299×10^{-3}	0.152500×10^{1}	0.802376×10^{-6}	0.809798×10^{0}
50	6.11	6.11000	0.26968×10^{-10}	0.152500×10^{1}	0.373477×10^{-9}	0.809800×10^{0}

Terminal penalties: $S_{11}=S_{22}=S_{33}=30{,}000$.
Prescribed states at terminal time: $x_{1_f}=1.525, x_{2_f}=0.0, x_{3_f}=0.8098$.

Table 4 Travel time to mean Martian orbital distance (in days)

Investigations	Solar sail spacecraft: For β_1 ($\alpha=0.1$ cm/s^2)	For β_2 ($\alpha=0.2$ cm/s^2)	Ion engine spacecraft
Tsu (logarithmic spiral)	250	118	
London (logarithmic spiral)	247		
Kelley (gradient)	412.5		
Kelley	412.5 (Euler)		197 (gradient)
Zhukov and Lebedev (indirect variational method)	405	322	
Present analysis (conjugate gradient)	445.5	355.4	
Pinkham, et al. (gradient)			195.4
Taylor, et al. (epsilon technique)			197
Jayaraman, et al. (conjugate gradient)			197
Stuhlinger (analytical)			222

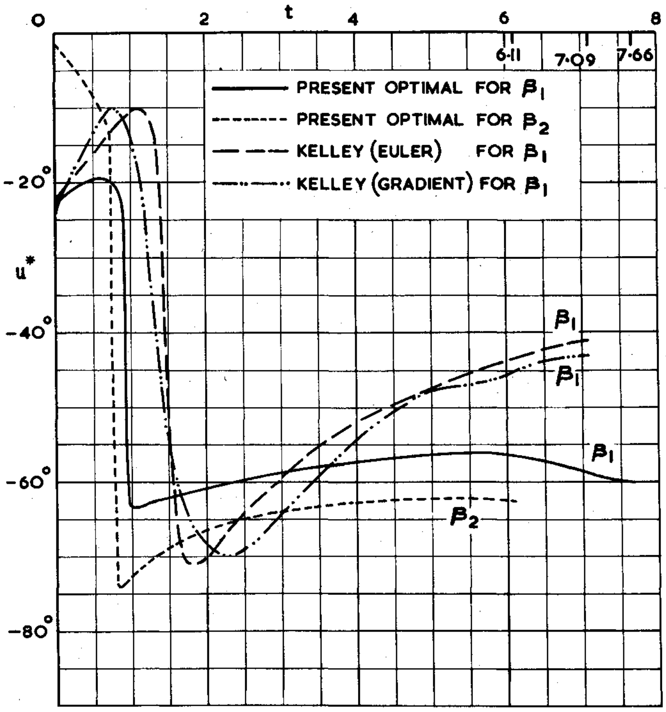

Fig. 2 Temporal evolution of optimal sail steering program.

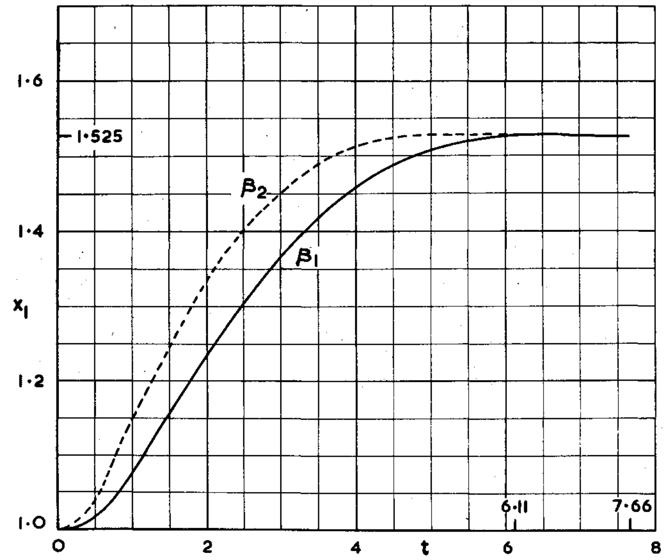

Fig. 3 Temporal evolution of radial displacement.

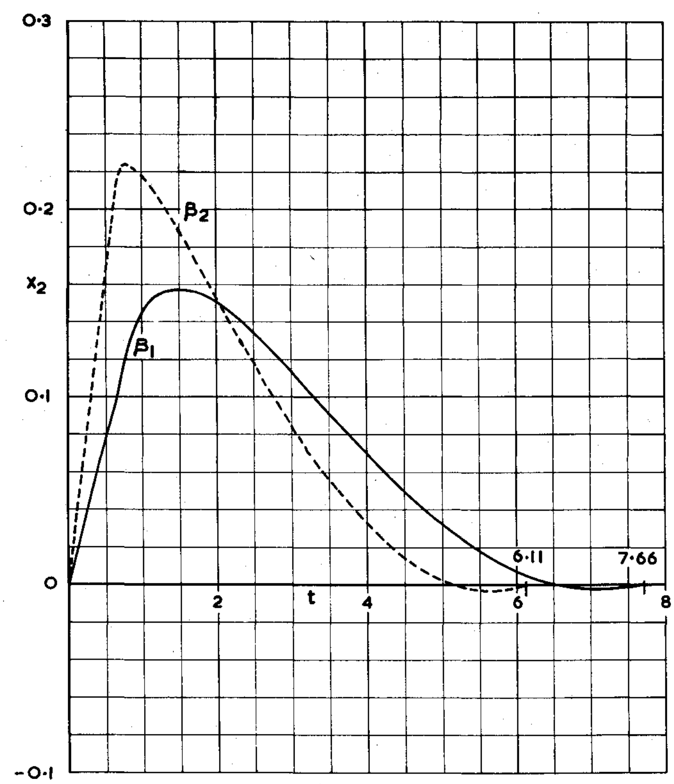

Fig. 4 Temporal evolution of radial velocity.

method of steepest descent. In the present work, as the convergence to the optimal solution is slower, it is felt that the terminal penalties are rather heavy; i.e., the tolerances employed for the reduction of errors in terminal values are "tight." Kelley also obtained an Euler solution satisfying the specified boundary conditions, employing an iterative technique due to Hestenes,[19] and he compared the gradient and Euler solutions. In Fig. 2, the optimal sail steering programs estimated by Kelley by employing gradient and Euler solutions have been shown along with that of the present work. By comparison with the Euler solution, it is seen that the departures of the control variable program (corresponding to β_1) due to the present work are more significant than that of the gradient solution. It may be mentioned here that Kelley assumed an explicit form for the control law as $u=a_0+a_1t+a_2t^2$, which is a second-order polynomial in t time where a_0, a_1, and a_2 are determined using the gradient technique. In the present formulation the optimal control history is obtained directly through numerical iteration of the conjugate gradient algorithm without any such restriction on the form of the control law. Hence, it is quite natural that the optimal control histories are different, which in turn would cause a difference in the performance functional. Also, it is worth mentioning here that there will always be a tradeoff between the penalty coefficient and the performance index, which is also apparent from the formulation of the problem in the present analysis. By means of such tradeoff studies with various choices of the set of terminal penalties, it is possible to achieve an optimized performance index closer to that obtained by Kelley and others, but with a loss of accuracy in the terminal conditions on the states. Kelley's control variable program seems to represent a "near-minimal" gradient solution, while that of the present analysis appears to be one of a family of neighboring extremals.

Tsu and London obtained the optimum travel time solutions along appropriate logarithmic spiral trajectories by disregarding the two-point boundary conditions. Thus, the results obtained by them do not represent "near-minimal" solutions of the Euler-Lagrange equations[15] governing optimal flight. Hence, the optimum travel time solutions obtained by them are not realizable without executing suitable thrust maneuvers with the aid of a chemical rocket engine so as to satisfy the initial conditions required to start the spacecraft along the optimum logarithmic spirals from the heliocentric orbit of Earth, as well as to match the kinematics of the spacecraft at the terminal time with that of the target planet in its heliocentric orbit. Moreover, it is worth mentioning here that the logarithmic spiral trajectories of solar sail spacecraft require a larger ΔV impulse than the two-impulse ballistic transfer along a Hohmann-transfer ellipse of the same duration. It may also be pointed out that the cotangential elliptic trajectories, which are generated by radial orientation of the solar sail having the appropriate value of the required sail acceleration, would cause a longer flight duration as compared to that along a Hohmann-transfer ellipse for travel to the mean orbital distance of the

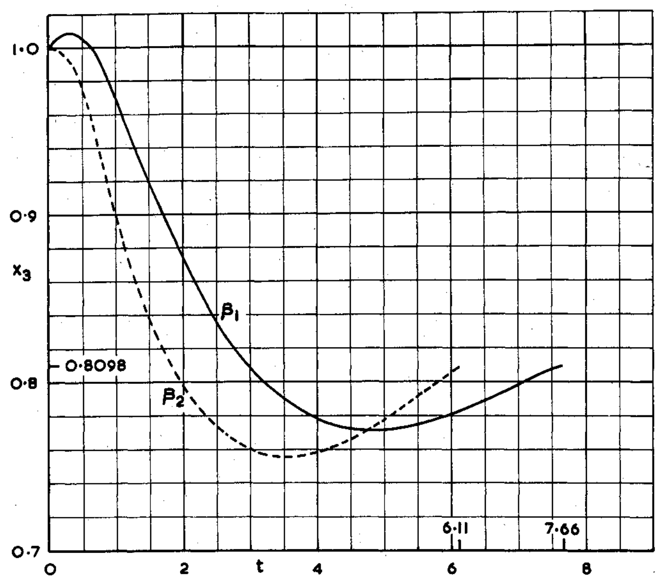

Fig. 5 Temporal evolution of circumferential velocity.

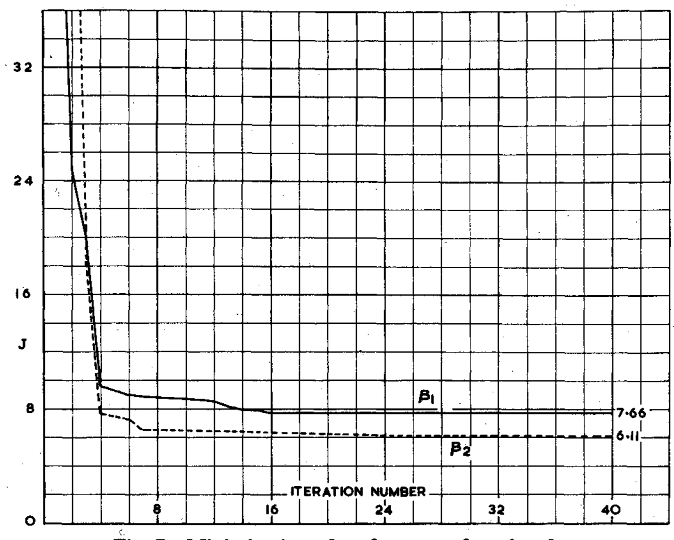

Fig. 7 Minimization of performance functional.

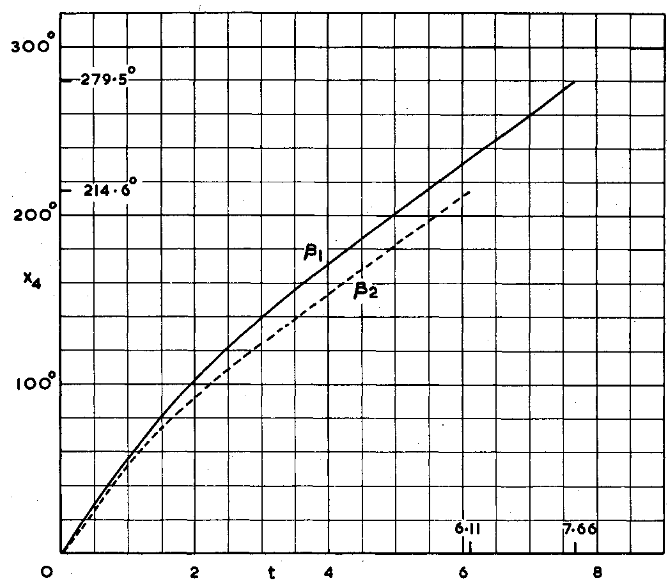

Fig. 6 Temporal evolution of heliocentric angle (terminal heliocentric angle open).

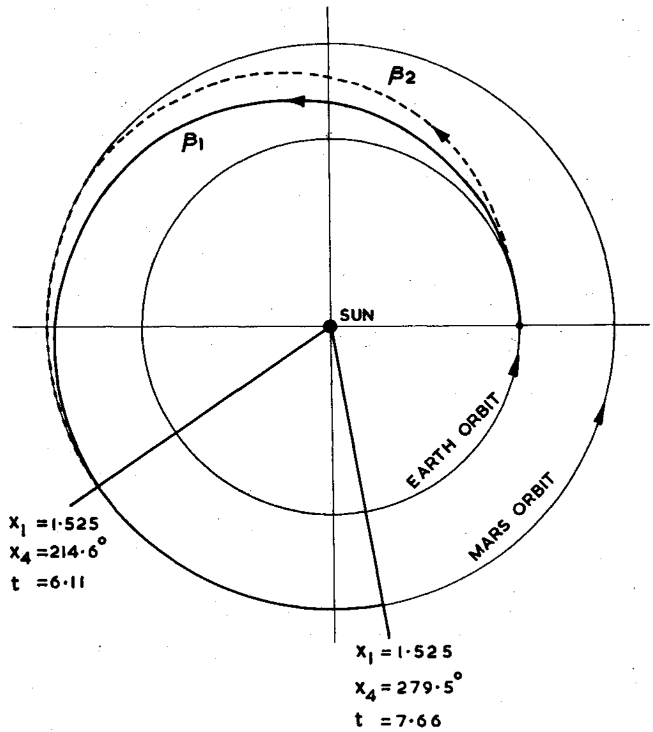

Fig. 8 Time-optimal orbit transfer trajectory (terminal heliocentric angle open).

target planet. Also, the ΔV impulse required at the aphelion of the cotangential elliptic trajectory for the solar sail would be larger than that required at the aphelion of the Hohmann-transfer ellipse. Moreover, if faster trips are attempted by employing sail accelerations in excess of those required for cotangential transfer, there would be a further increase in the ΔV impulse upon arrival at the orbit of the target planet. Thus, neither the logarithmic spiral trajectories nor the conic section trajectories are of interest from the standpoint of the most efficient utilization of the solar sail.

Travel Time Comparison with Ion Engine Spacecraft

Feasibility studies on the ionic propulsion system were originally carried out by Stuhlinger in connection with the problem of Earth-Mars orbit transfer. He approached the problem from the analytical standpoint without placing emphasis on the minimization of energy or time of the trajectory. Kelley and several other investigators formulated this problem from the viewpoint of optimal control and obtained the minimum travel time solution, assuming numerical estimates for the thrust and mass parameters of the ion engine from the work of Edwards and Brown,[20] employing various minimization algorithms. The solutions obtained by them are shown in Table 4. A perusal of Table 4 reveals that the minimum travel time solutions for the ion engine reported by Kelley as well as other investigators are in close agreement, and that Pinkham, et al.,[12] have obtained the smallest final time of 195.4 days for executing the orbit transfer to Mars using the ion engine spacecraft.

Based upon estimates of the attainable acceleration levels quoted in the published literature corresponding to existing technology of the solar sail and electric propulsion systems, a comparison of the results presented in Table 4 from the standpoint of minimum travel time to mean Martian orbital distance reveals the following: Zhukov and Lebedev, by employing a sail acceleration of 0.1 cm/s^2, obtained the smallest final time of 405 days (less than that due to Kelley and the present analysis), which is about 2.1 times that required by the ion engine spacecraft. Also, for a sail acceleration of 0.2 cm/s^2, they arrived at a minimum travel time of 322 days (less than that due to the present analysis), which is about 1.7 times that required by the electric propulsion vehicle. Further, even for a sail acceleration of 0.5 cm/s^2, which is very much higher than the initial thrust acceleration of 0.083 cm/s^2 (due to Edwards and Brown) developed by the ion engine, Zhukov and Lebedev have obtained a minimum travel time of about 250 days which is about 1.3 times that required by the ion engine.

It is obvious that the solar sail spacecraft, with the attainable acceleration levels quoted in the published literature on sail technology, does not appear to be on a par with the electric propulsion system from the standpoint of minimum travel time. Using other terminology, execution of faster trips involving less travel duration than the "near-minimal" travel time solutions to the Martian orbit by means of solar sail would call for the incorporation of additional onboard propulsive devices. Thus, without the employment of additional propulsive devices for executing the necessary thrust maneuvers, the solar sail spacecraft does not appear to be a faster mode of transport in comparison with the ion engine spacecraft for voyage to Mars.

Conclusions

Based on the optimal sail steering programs obtained in the present analysis and the subsequent discussions, the following conclusions may be drawn:

1) The switching instant for the optimal sail steering program decreases with increase in sail acceleration.

2) Prior to the switching instant, the optimal sail setting is closer to radial orientation for every sail acceleration; and for a higher sail acceleration, the sail setting is nearer to the radial orientation as compared to that for a lower sail acceleration, thus giving rise to larger outward radial acceleration of the spacecraft.

3) After the switching instant, the optimal sail setting is closer to the local horizon for every sail acceleration; and for a higher sail acceleration, the sail setting is nearer to the local horizon as compared to that for a lower sail acceleration.

4) For every sail acceleration, the optimal sail setting remains closer to the local horizon for a greater part of the total optimized orbit transfer time.

5) The optimized orbit transfer time decreases with increase in sail acceleration.

Finally, through a comparison of the "near-minimal" travel time solutions for the solar sail and ion engine spacecraft reported in the published literature and the present analysis, it may be concluded that without the employment of additional onboard propulsive devices, the solar sail spacecraft does not appear to be a faster mode of transport in comparison with the electric propulsion system for voyage to Mars.

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